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On the anelastic properties of materials containing gas-filled grain boundary bubbles

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Models for the growth of grain boundary voids may be extended to describe the strain-time response of a stressed material containing an array of gas-filled grain boundary bubbles. Above a critical value of applied stress the bubbles grow indefinitely until failure of the material occurs. Below this critical value significant anelastic strain may be observed following a stress change. The strain is not easily formulated explicitly in terms of time but may be evaluated using numerical techniques.

1. Introduction

The stress-assisted growth of grain boundary voids arising from the ability of the boundaries to act as perfect sources and sinks for vacancies has been the subject of detailed and comprehensive analysis in the literature. See, for example, the review by Cocks and Ashby [1]. In most cases the differential equation describing the rate of void growth takes the form

$$\partial V/\partial t = KF \quad (1)$$

where F is a driving force for growth and K a parameter which for example [1] may take the form

$$K = 2\pi\Omega w D_g/kT \quad (2)$$

whilst a more detailed analysis [2] in which the increase in volume of the cavities due to the jacking apart of adjacent grains during creep has been considered leads to

$$K = 2\pi\Omega w D_g/kT\{\ln(l/r) - [1 - (r^2/l^2)][3 - (r^2/l^2)]/4\} \quad (3)$$

where D_g is the grain boundary self diffusivity, w the boundary width, and Ω the atomic volume. The void spacing is $2l$ and the void radius is r. F which is a term describing the driving force for void growth consists of the applied stress σ and a second term $-2\gamma/r$ arising from the surface tension γ which opposes such growth [2].

In this paper we consider the case where the cavity is initially a bubble of radius r_0 containing gas at a pressure P which balances the surface tension such that with no applied stress

$$P = 2\gamma/r_0 \quad (4)$$

Such bubbles may occur by diffusion of gas through the material or in nuclear materials from internal fission gas. The pressure term must be included in the expression for the driving force for bubble growth so that the full expression is

$$F = \sigma - (2\gamma/r) + P(r) \quad (5)$$

where $P(r)$ is the pressure as a function of the bubble radius r. Assuming that the ideal gas law holds, this function may be written as

$$P(r) = (2\gamma/r_0)(r_0/r)^3 \quad (6)$$

so that Equation 5 becomes

$$F = \sigma - (2\gamma/r_0)[r_0/r - (r_0/r)^3] \quad (7)$$

This driving force is shown plotted in normalized form against bubble radius in Fig. 1 from which it is noted that for all compressive stresses and for low tensile stresses the curve crosses the horizontal axis. However above a critical value σ^* a minimum growth rate is reached but the curve does not become negative and the bubble grows indefinitely until specimen failure occurs. Differentiating Equation 7 to find the minimum F^* leads [3] to

$$F^* = \sigma - (2\gamma/r_0)(2/3\sqrt{3}) \quad (8)$$

The critical stress is when F^* is zero

$$\sigma^* = (2\gamma/r_0)(2/3\sqrt{3}) \quad (9)$$

Three distinct types of response to an applied stress may be described with reference to Fig. 1: (i) $\sigma < 0$ (ii) $0 < \sigma < \sigma^*$ (iii) $\sigma > \sigma^*$. In case (i) the bubble is initially in equilibrium at point **a** with no applied stress and radius r_0. A compressive stress is then applied instantaneously along **ab** and the bubble then shrinks progressively along **bc** according to Equation 1. At point **c** equilibrium is once more achieved and the bubble ceases to change in volume. If the stress is then instantaneously removed along **cd** the bubble grows along **da** until the original radius is reached at point **a**. The corresponding "circuit" **aefg** is indicated for tension (case (ii)) in the same figure. If, however, a supercritical tensile stress is applied to point **h** (case

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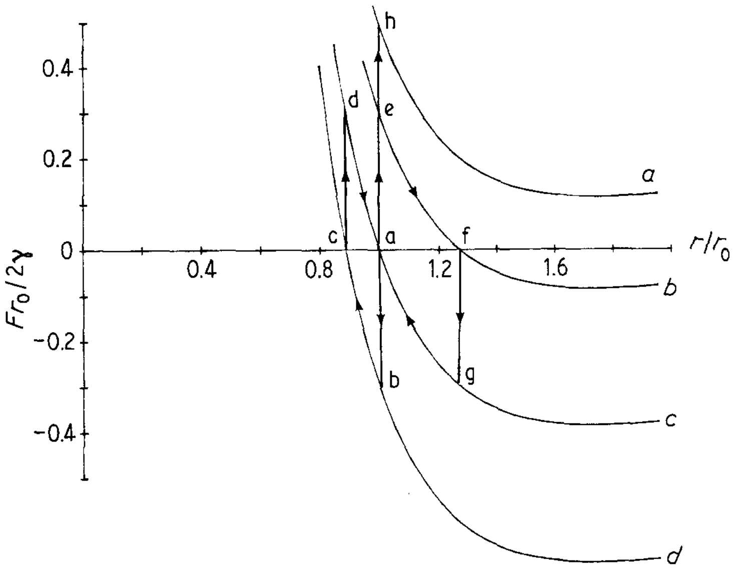

Figure 1 Equation 7 plotted in normalized form for various stress levels, $\sigma r_0/2\gamma$ (*a* 0.5, *b* 0.3, *c* 0, *d* −0.3).

(iii)) then the bubble continues to grow until failure of the material occurs.

2. Analytical approach

In order to obtain quantitative information on the strain-time response of a material containing gas-filled grain boundary bubbles, consider the case where identical bubbles are arranged in a simple array on the boundaries. Let $l \gg r$ and let the grain size be d. The specimens strain is given by

$$\varepsilon = 4\pi(r^3 - r_0^3)/3dl^2 \quad (10)$$

Differentiating with respect to the bubble radius

$$\partial\varepsilon/\partial r = 4\pi r^2/dl^2 \quad (11)$$

The strain rate in compression is given by

$$\partial\varepsilon/\partial t = (\partial\varepsilon/\partial r)(\partial V/\partial t)(\partial r/\partial V) = (K/dl^2)\{-\sigma - (2\gamma/r_0)[(r_0/r) - (r_0/r)^3]\} \quad (12)$$

Substituting K from Equation 2 and writing $f(\sigma) = -\sigma - (2\gamma/r_0)[(r_0/r) - (r_0/r)^3]$ leads to

$$\partial\varepsilon/\partial t = 2\pi\Omega w D_g f(\sigma)/dl^2kT \quad (13)$$

Then writing $\Omega \sim b^3$ and $w \sim 2b$ where b is Burger's vector and introducing the shear modulus G as a stress normalizing factor

$$\partial\varepsilon/\partial t = (4\pi D_g Gb/kT)(b/d)(b/l)^2(f(\sigma)/G) \quad (14)$$

This expression has the same general form as the Dorn equation and is shown plotted for different values of σ (r_0 constant) and r_0 (σ constant) in Figs 2 and 3, respectively. An appropriate value of D_g was taken from Brown and Ashby [4]. For low values of r/r_0 the bubble radius has the most influence on the strain rate whereas for higher values of r/r_0 the value of the applied stress is more significant.

To obtain the strain-time response of the material it is required to write Equation 12 in terms of strain. By substituting from Equation 10

$$\partial\varepsilon/\partial t = K\{\sigma - 2\gamma[(1 + 3\varepsilon dl^2/4\pi r_0^3)^{-1/3} - (1 + 3\varepsilon dl^2/4\pi r_0^3)^{-1}]/r_0\}/dl^2 \quad (15)$$

The integration of Equation 15 is intractable and therefore numerical techniques are required to predict strain-time behaviour.

3. Numerical analysis

For the numerical analysis the more exact form of the bubble growth equation due to Speight and Beere [2] was used. The controlling differential equation was,

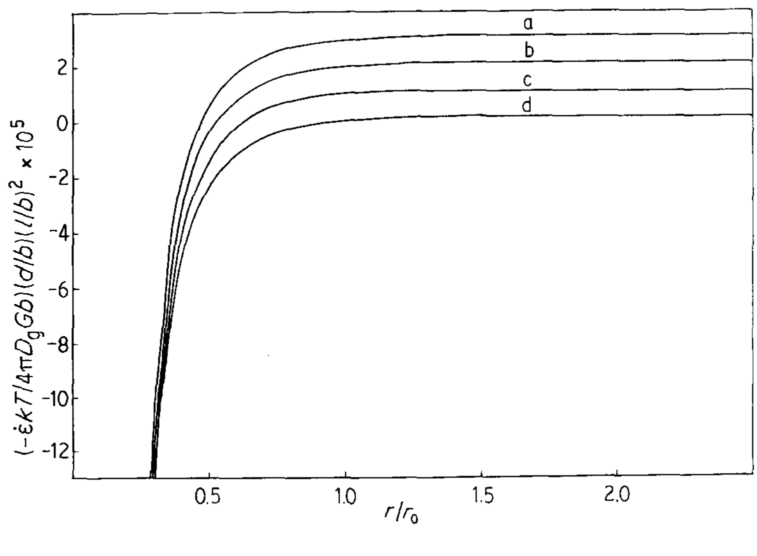

Figure 2 Equation 13 plotted for various values of stress (a 3 MPa, b 2 MPa, c 1 MPa, d no applied stress). The initial bubble radius is 1 μm.

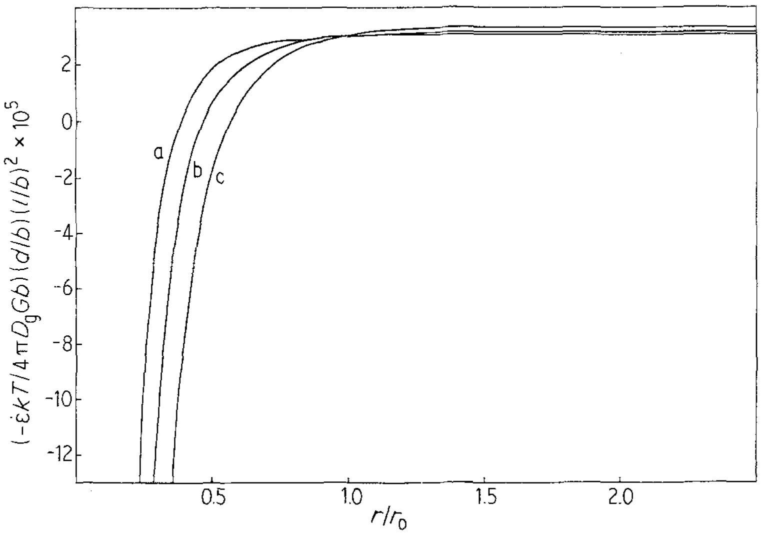

Figure 3 Equation 13 plotted for various values of initial bubble radius (a $2\mu m$, b $1\mu m$, c $0.5\mu m$). There is no applied stress.

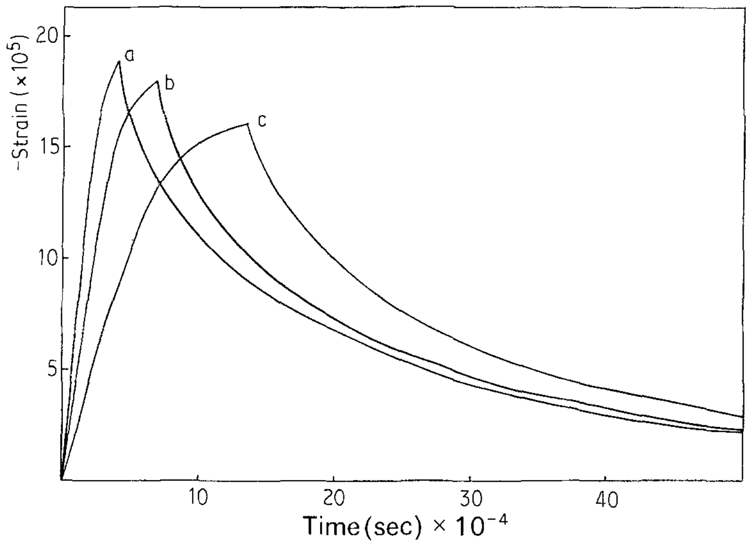

Figure 4 Strain-time plots for anelasticity due to grain boundary bubbles for several values of stress (a 3 MPa, b 2 MPa, c 1 MPa) as predicted for compression by the numerical analysis.

therefore, formed by combining Equations 1, 3 and 5, noting that for compression the value of σ is negative. In a time interval δt the bubble shrinks by a volume δV given by this equation. A corresponding volume of vacancies is absorbed by the grain boundary which is assumed to act as a perfect sink. Assuming a square array on the boundary this leads to a strain increment of

$$\delta\varepsilon = \delta V/d(l^2 - \pi r^2) \qquad (16)$$

The process is repeated until point **c** is reached (Fig. 1); that is when $\delta V = 0$. The magnitude and time dependence of the anelastic effect are shown in Fig. 4 for different values of stress. Quite large anelastic strains are produced in compression. Fig. 5 shows the dependence of the maximum strain on (a) the applied stress and (b) the bubble spacing. Only at quite low stresses is the maximum strain strongly dependent on the stress. As the bubble spacing decreases the maximum strain increases rapidly as a larger volume fraction of the material becomes composed of compressible gas.

For tension there is only a limiting value of r/r_0

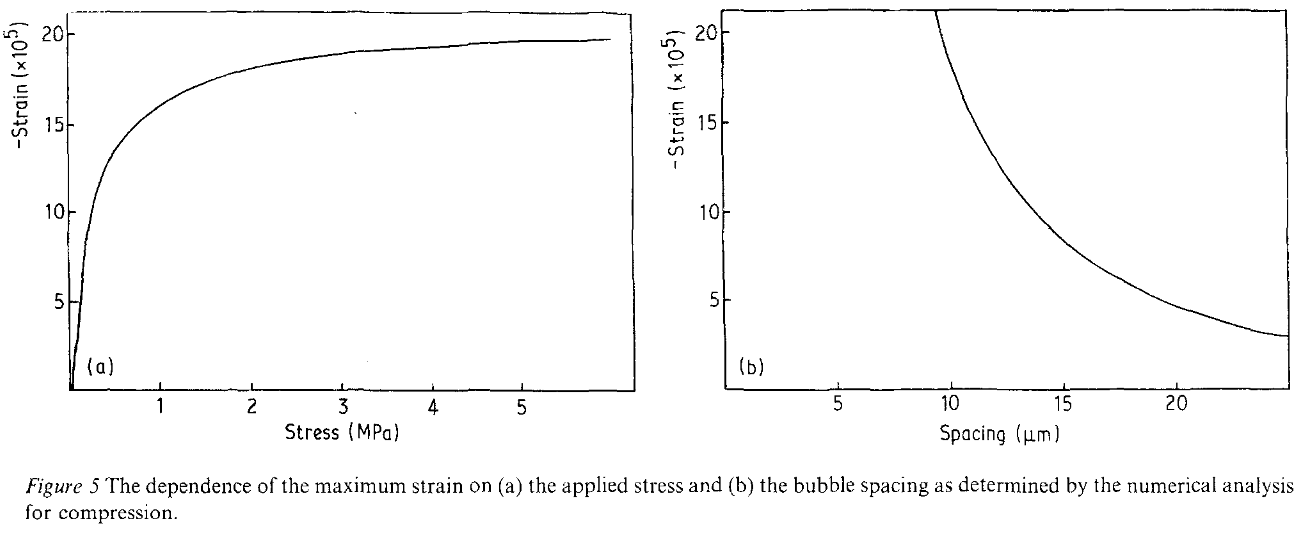

Figure 5 The dependence of the maximum strain on (a) the applied stress and (b) the bubble spacing as determined by the numerical analysis for compression.

when the applied stress has a value below σ^*. Thus the stresses involved in this case and hence the magnitude of the anelastic strains are much smaller than for compression.

4. Conclusions

Well established models for the growth of grain boundary voids have been used to demonstrate that significant anelastic strains can result in materials containing arrays of gas-filled grain boundary bubbles. The strain-time response of such a material cannot easily be derived analytically but may be evaluated using numerical techniques.

For compressive stresses quite large anelastic strains can occur, the maximum strain being sensitive to stress and the bubble spacing. For tension an applied stress above a certain critical value leads to continuous bubble growth which eventually results in failure of the material. Below this critical value anelastic strains may be observed but these are smaller than for compression due to the lower stress involved.

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